Describing Data: Graphs and Tables

Basic Concepts

Frequency Tables and Histograms

Bar and Pie Charts

Scatter Plots

Time Series Plots

Basic Concepts in Data Analysis

Data, Information, and Knowledge

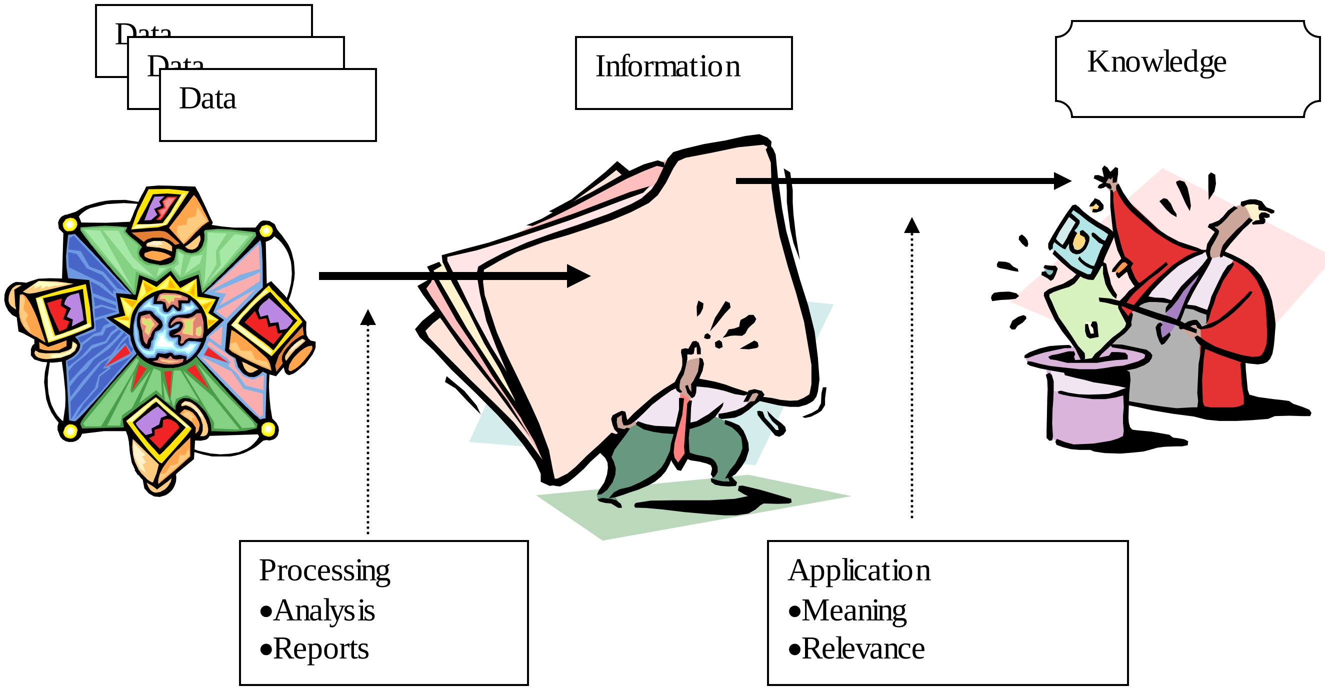
Populations and Samples

Variables and Observations

Types of Data: Categorical and Numerical

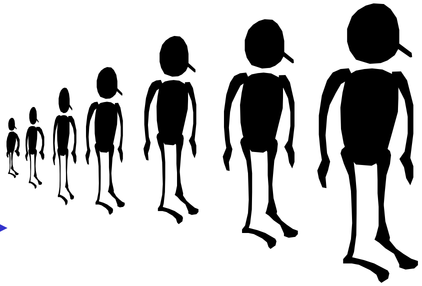
Types of Data: Cross Sectional and Time Ordered

Data, Information, and Knowledge



Populations and Samples

Statistical Inference



Sample: Subset of collection of all possible entities (observation units)

Data on sample is what is available.

KNOWN
Statistics are used to describe samples. These can vary across samples.

Statistical Inference *is the art and science of drawing inferences/ conclusions about a population of interest.*

Population: Collection of all possible entities (observation units)

Data on the whole population is usually not available.

UNKNOWN
Parameters are used to describe populations. These are constants for a population.

Variables and Observations

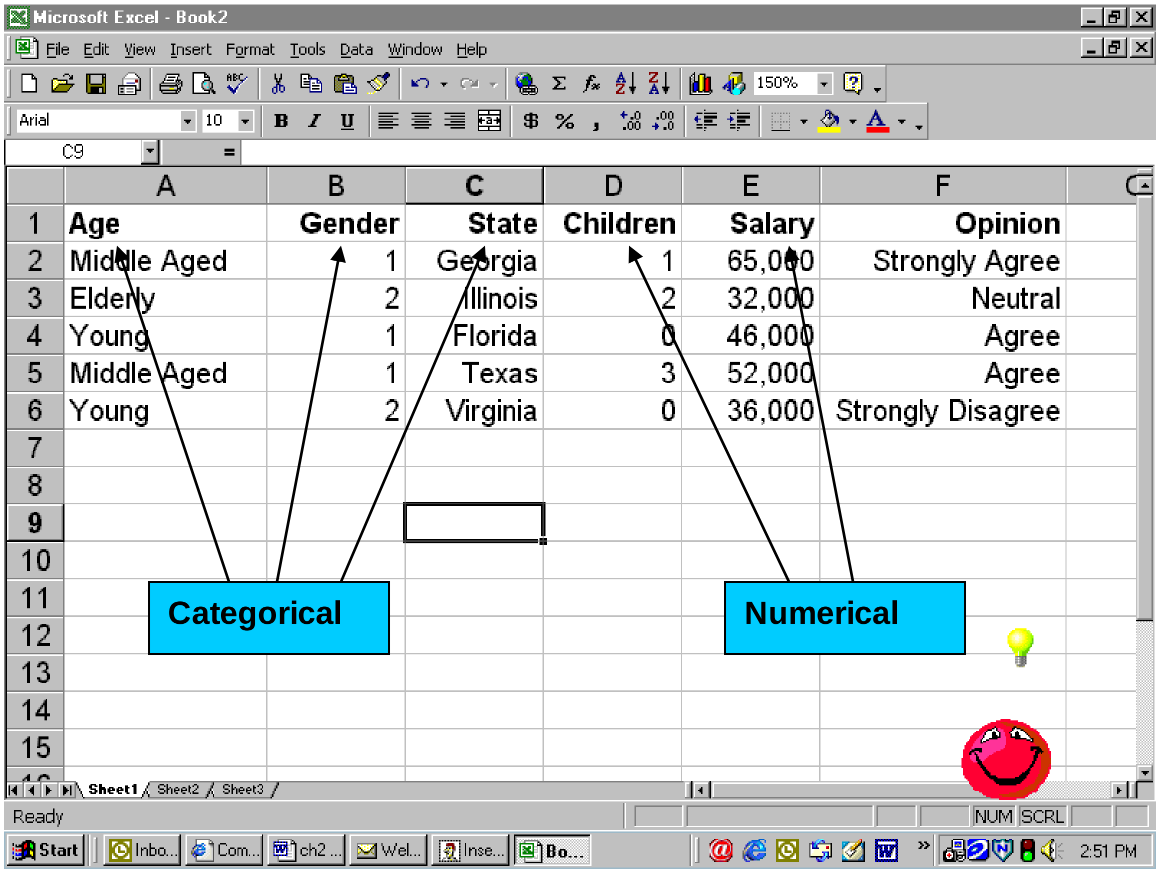
VARIABLES

OBSERVATIONS

Entity	**Height** (inches)	**Weight** (pounds)	**Age** (years)	**Sex** (Category)
Person 1	67	170	33	Male
Person 2	61	120	38	Female
Person 3	72	220	62	Male
*	*	*	*	*
*	*	*	*	*

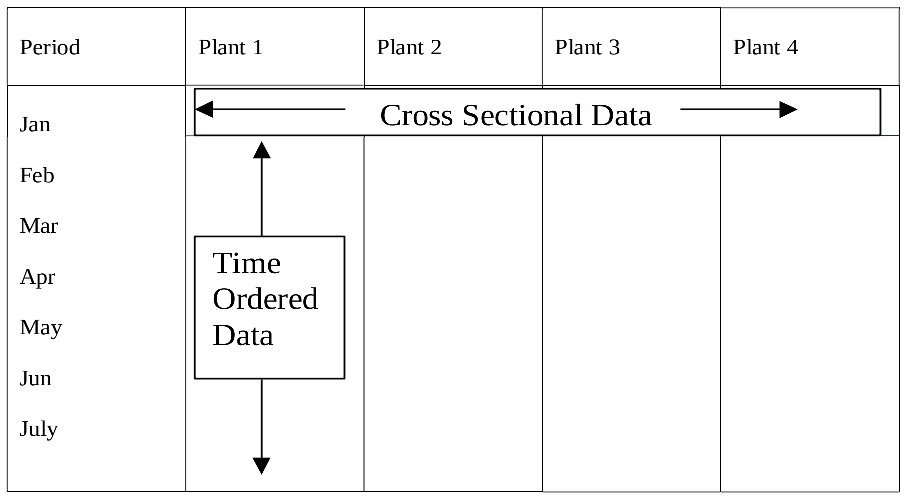
Measurement

Types of Data: Categorical and Numerical



Age	Gender	State	Children	Salary	Opinion
Middle Aged	1	Georgia	1	65,000	Strongly Agree
Elderly	2	Illinois	2	32,000	Neutral
Young	1	Florida	0	46,000	Agree
Middle Aged	1	Texas	3	52,000	Agree
Young	2	Virginia	0	36,000	Strongly Disagree

Types of Data: Cross-sectional and Time Ordered



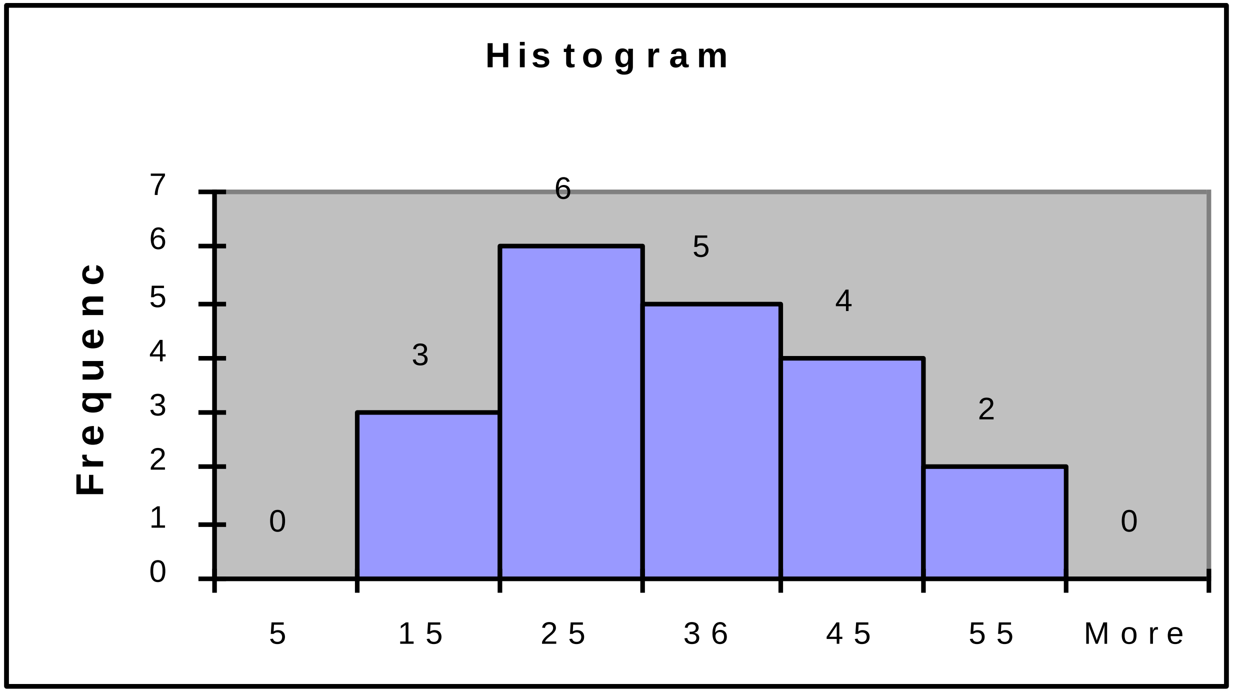
Frequency Tables

A Frequency Table showing a classification of the AGE of attendees at an event.

Class	Frequency	Relative Frequency	Percentage
10 but under 20	3	.15	15
20 but under 30	6	.30	30
30 but under 40	5	.25	25
40 but under 50	4	.20	20
50 but under 60	2	.10	10
Total	20	1	100

Frequency Histograms

A graphical display of distribution of frequencies



Developing Frequency Tables and Histograms

Sort Raw Data in Ascending Order:

12, 13, 17, 21, 24, 24, 26, 27, 27, 30, 32, 35, 37, 38, 41, 43, 44, 46, 53, 58

Find Range: 58 - 12 = 46

Select Number of Classes: 5 (usually between 5 and 15)

Compute Class Interval (width): 10 (range/classes = 46/5 then round up)

Determine Class Boundaries (limits): 10, 20, 30, 40, 50

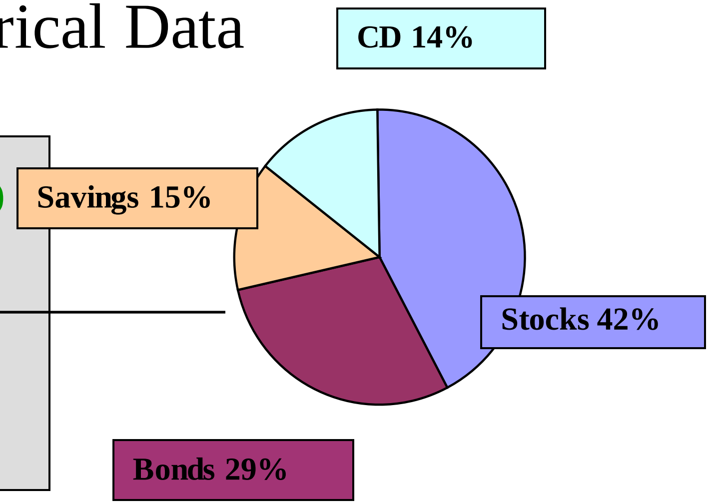
Compute Class Midpoints: 15, 25, 35, 45, 55

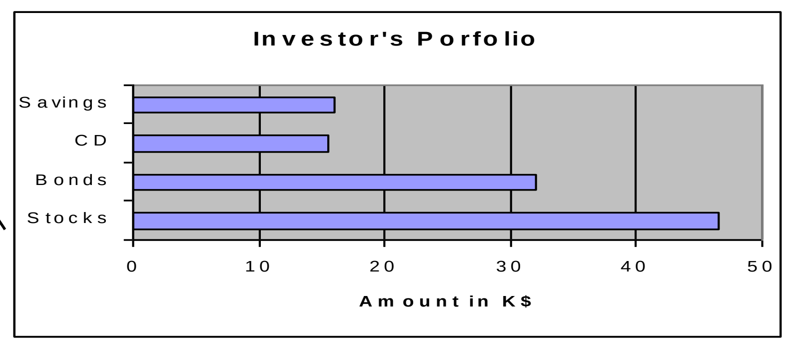
Count Observations & Assign to Classes

Bar and Pie Charts

Displaying Categorical Data

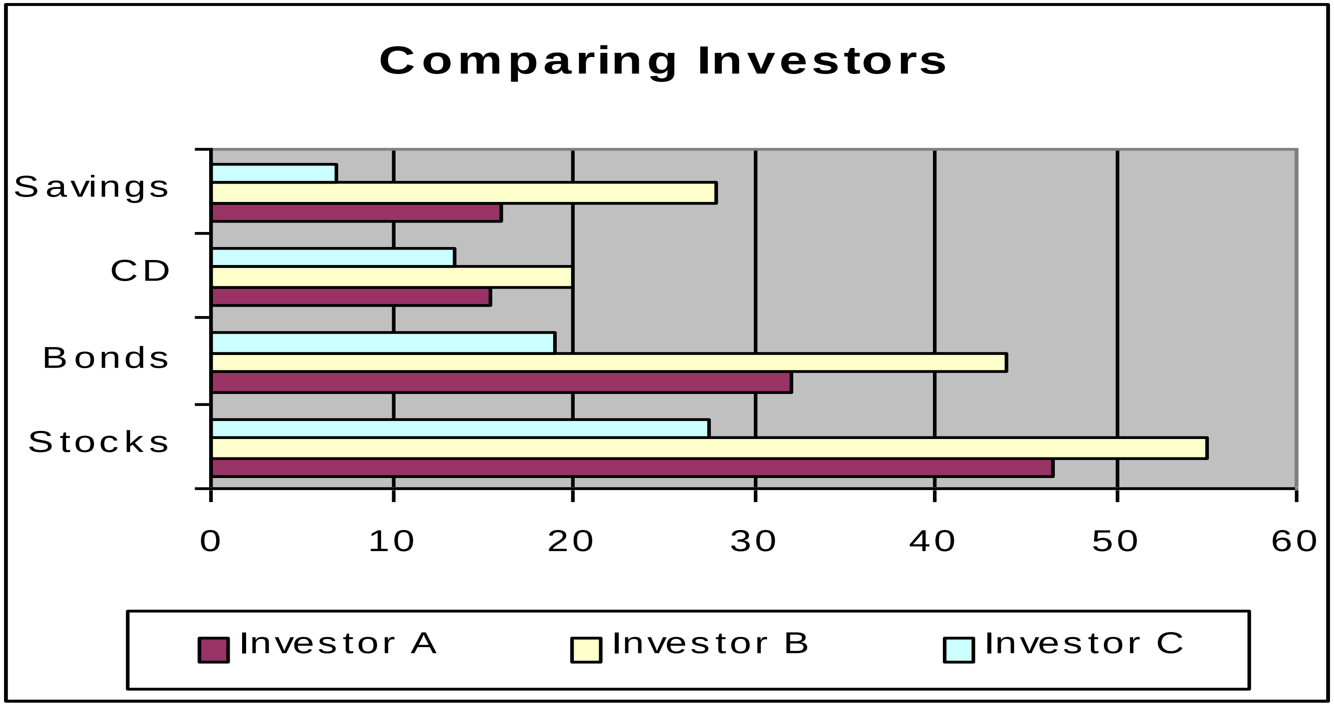
Investment Category	Amount	Percentage (in thousands $)
Stocks	46.5	42.27
Bonds	32	29.09
CD	15.5	14.09
Savings	16	14.55
Total	**110**	**100**





Side by Side Chart

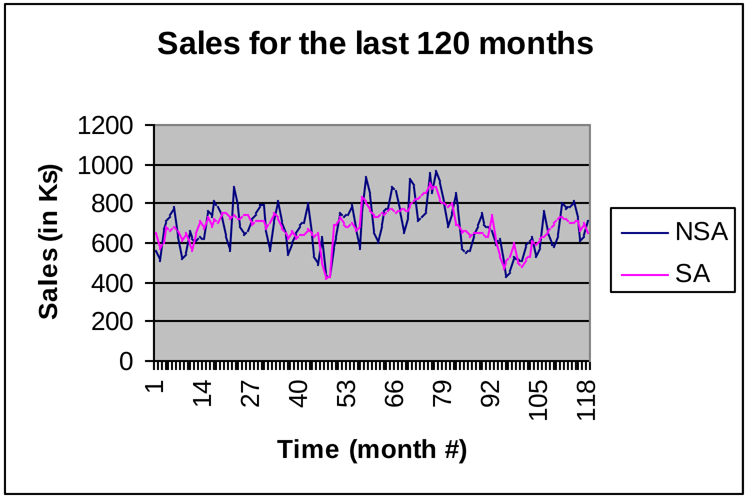
Displaying Categorical Bivariate Data: Contingency Tables and Side-by-Side Charts



Scatter Plot for bivariate numerical data

Shows relationship between two variables.

Can one be used to predict the other?



Time-Series and Regression Analysis are used to predict one variable's value based on the other. Correlation analyses is used to measure the strength of linear relationship among two variables.

Descriptive Statistics

Basic Concepts

Frequency Tables and Histograms

Bar and Pie Charts

Scatter Plots

Time Series Plots

Learning Objectives

- Distinguish between measures of central tendency, measures of variability, and measures of shape
- Understand the meanings of mean, median, mode, quartile, percentile, and range
- Compute mean, median, mode, percentile, quartile, range, variance, standard deviation, and mean absolute deviation

Learning Objectives -- Continued

- Differentiate between sample and population variance and standard deviation
- Understand the meaning of standard deviation as it is applied by using the empirical rule
- Understand box and whisker plots, skewness, and kurtosis

Measures of Central Tendency

- Measures of central tendency yield information about “particular places or locations in a group of numbers.”
- Common Measures of Location
  - Mode
  - Median
  - Mean
  - Percentiles
  - Quartiles

Mode

- The most frequently occurring value in a data set
- Applicable to all levels of data measurement (nominal, ordinal, interval, and ratio)

- Bimodal -- Data sets that have two modes
- Multimodal -- Data sets that contain more than two modes

Mode -- Example

- The mode is 44.
- There are more 44s than any other value.

35	41	44	45
37	41	44	46
37	43	44	46
39	43	44	46
40	43	44	46
40	43	45	48

Median

- Middle value in an ordered array of numbers.
- Applicable for ordinal, interval, and ratio data
- Not applicable for nominal data
- Unaffected by extremely large and extremely small values.

Median: Computational Procedure

- First Procedure
  - Arrange observations in an ordered array.
  - If number of terms is odd, the median is the middle term of the ordered array.
  - If number of terms is even, the median is the average of the middle two terms.

- Second Procedure
  - The median's position in an ordered array is given by (n+1)/2.

Median: Example with an Odd Number of Terms

Ordered Array includes:

3 4 5 7 8 9 11 14 15 16 16 17 19 19 20 21 22

- There are 17 terms in the ordered array.
- Position of median = (n+1)/2 = (17+1)/2 = 9
- The median is the 9th term, 15.
- If the 22 is replaced by 100, the median remains at 15.
- If the 3 is replaced by -103, the median remains at 15.

Mean

- Is the average of a group of numbers
- Applicable for interval and ratio data, not applicable for nominal or ordinal data
- Affected by each value in the data set, including extreme values
- Computed by summing all values in the data set and dividing the sum by the number of values in the data set

Population Mean

$$\mu = \frac{\sum X}{N} = \frac{X_1 + X_2 + X_3 + \ldots + X_N}{N}$$

$$= \frac{24 + 13 + 19 + 26 + 11}{5}$$

$$= \frac{93}{5}$$

$$= 18.6$$

Sample Mean

$$\bar{X} = \frac{\sum X}{n} = \frac{X_1 + X_2 + X_3 + \ldots + X_n}{n}$$

$$= \frac{57 + 86 + 42 + 38 + 90 + 66}{6}$$

$$= \frac{379}{6}$$

$$= 63.167$$

Quartiles

Measures of central tendency that divide a group of data into four subgroups

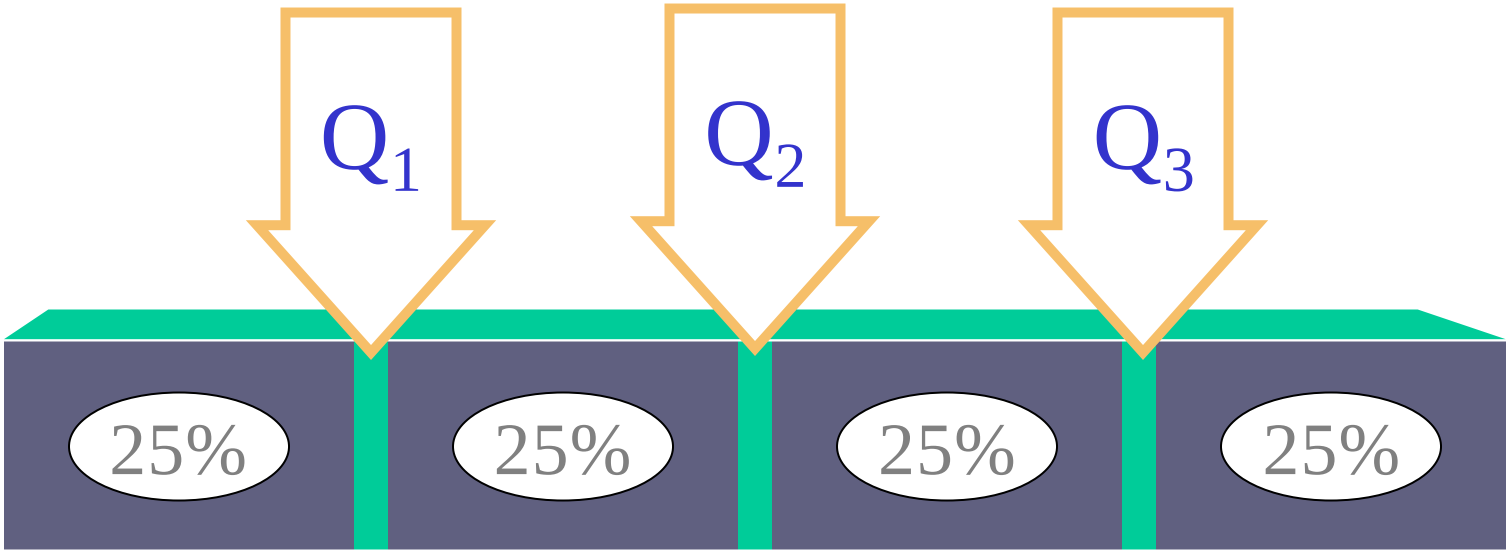
- Q_1: 25% of the data set is below the first quartile
- Q_2: 50% of the data set is below the second quartile
- Q_3: 75% of the data set is below the third quartile

Quartiles, *continued*

- Q_1 is equal to the 25th percentile
- Q_2 is located at 50th percentile and equals the median
- Q_3 is equal to the 75th percentile

Quartile values are not necessarily members of the data set

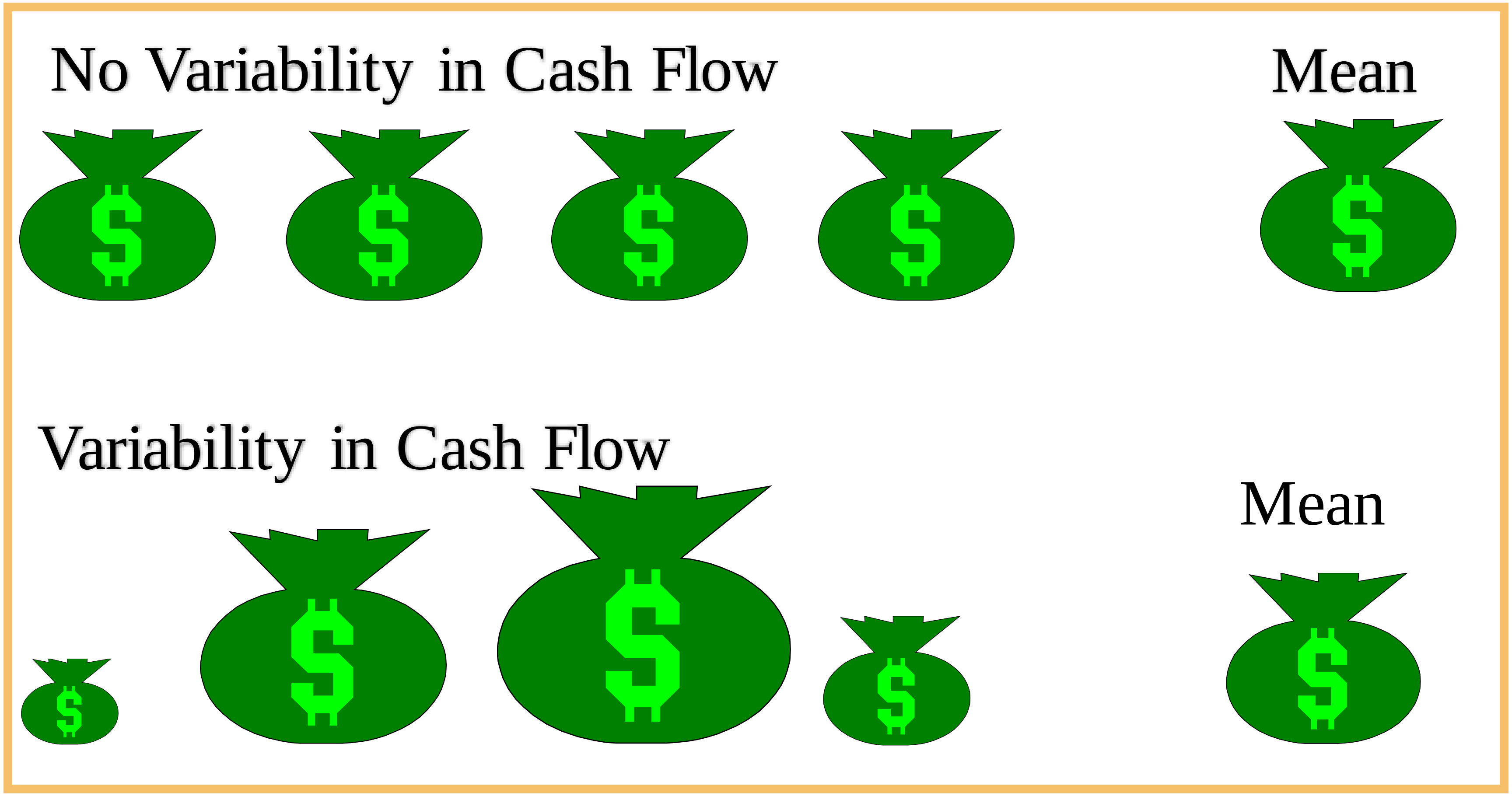
Quartiles



Measures of Variability

- Measures of variability describe the spread or the dispersion of a set of data.
- Common Measures of Variability
  - Range
  - Interquartile Range
  - Mean Absolute Deviation
  - Variance
  - Standard Deviation
  - Z scores
  - Coefficient of Variation

Variability



Range

- The difference between the largest and the smallest values in a set of data
- Simple to compute
- Ignores all data points except the two extremes
- Example:
- Range = Largest - Smallest
  48 - 35 = 13

35	41	44	45
37	41	44	46
37	43	44	46
39	43	44	46
40	43	44	46
40	43	45	48

Interquartile Range

- Range of values between the first and third quartiles
- Range of the “middle half”
- Less influenced by extremes

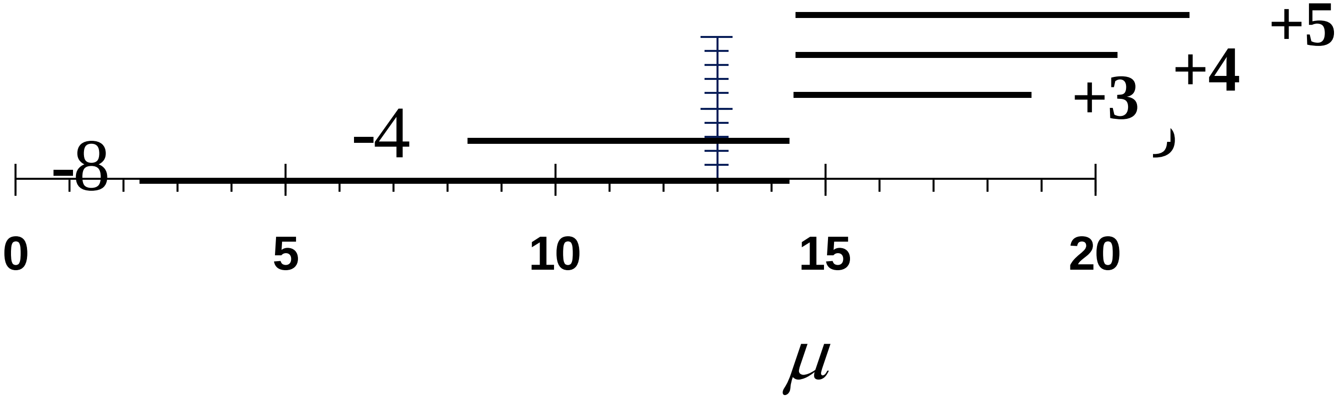
$$\boxed{Interquartile\ Range = Q_3 - Q_1}$$

Deviation from the Mean

- Data set: 5, 9, 16, 17, 18
- Mean:

$$\boxed{\mu = \frac{\sum X}{N} = \frac{65}{5} = 13}$$

- Devi



Mean Absolute Deviation

- Average of the absolute deviations from the mean

X	$X - \mu$	$\|X - \mu\|$
5	**-8**	**+8**
9	**-4**	**+4**
16	**+3**	**+3**
17	**+4**	**+4**
18	**+5**	**+5**
	0	**24**

$$M.A.D. = \frac{\sum |X - \mu|}{N}$$

$$= \frac{24}{5}$$

$$= 4.8$$

Population Variance

- Average of the squared deviations from the arithmetic mean

X	$X-\mu$	$(X-\mu)^2$
5	**-8**	**64**
9	**-4**	**16**
16	**+3**	**9**
17	**+4**	**16**
18	**+5**	**25**
	0	**130**

$$\sigma^2 = \frac{\sum(X-\mu)^2}{N}$$

$$= \frac{130}{5}$$

$$= 26.0$$

Population Standard Deviation

- Square root of the variance

X	$X-\mu$	$(X-\mu)^2$
5	-8	64
9	-4	16
16	+3	9
17	+4	16
18	+5	25
	0	130

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N} = \frac{130}{5} = 26.0$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{26.0} = 5.1$$

Sample Variance

- Average of the squared deviations from the arithmetic mean

X	$X - \overline{X}$	$\left(X - \overline{X}\right)^2$
2,398	**625**	**390,625**
1,844	**71**	**5,041**
1,539	**-234**	**54,756**
1,311	**-462**	**213,444**
7,092	**0**	**663,866**

$$S^2 = \frac{\sum \left(X - \overline{X}\right)^2}{n-1} = \frac{663,866}{3} = 221,288.67$$

Sample Standard Deviation

- Square root of the sample variance

X	$X - \bar{X}$	$(X - \bar{X})^2$
2,398	625	390,625
1,844	71	5,041
1,539	-234	54,756
1,311	-462	213,444
7,092	0	663,866

$$S^2 = \frac{\sum (X - \bar{X})^2}{n-1} = \frac{663{,}866}{3} = 221{,}288.67$$

$$S = \sqrt{S^2} = \sqrt{221{,}288.67} = 470.41$$

Coefficient of Variation

- Ratio of the standard deviation to the mean, expressed as a percentage
- Measurement of relative dispersion

$$\boxed{C.V. = \frac{\sigma}{\mu}(100)}$$

Coefficient of Variation

$$\mu_1 = 29$$

$$\sigma_1 = 4.6$$

$$C.V._1 = \frac{\sigma_1}{\mu_1}(100)$$

$$= \frac{4.6}{29}(100)$$

$$= 15.86$$

$$\mu_2 = 84$$

$$\sigma_2 = 10$$

$$C.V._2 = \frac{\sigma_2}{\mu_2}(100)$$

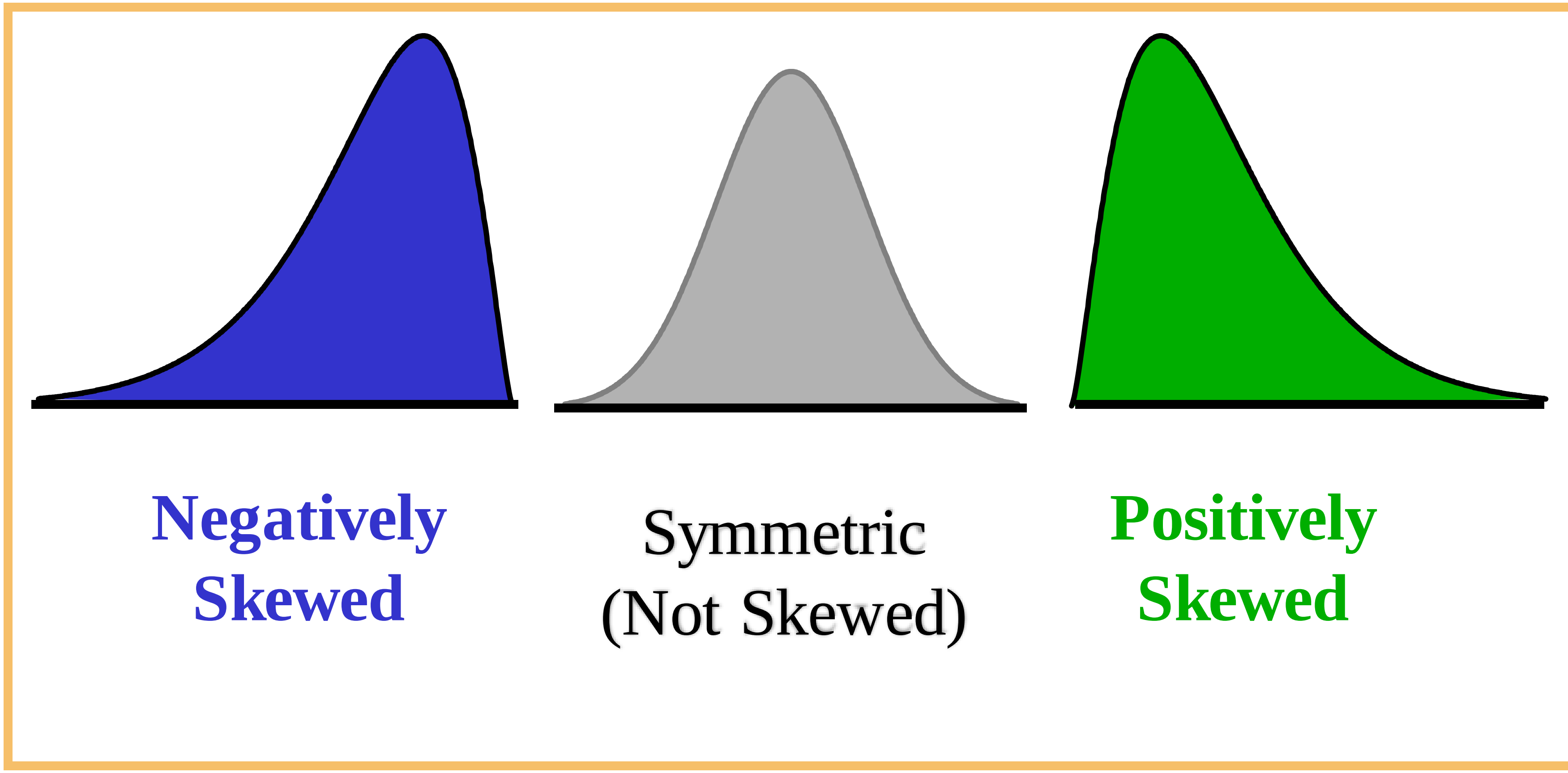
$$= \frac{10}{84}(100)$$

$$= 11.90$$

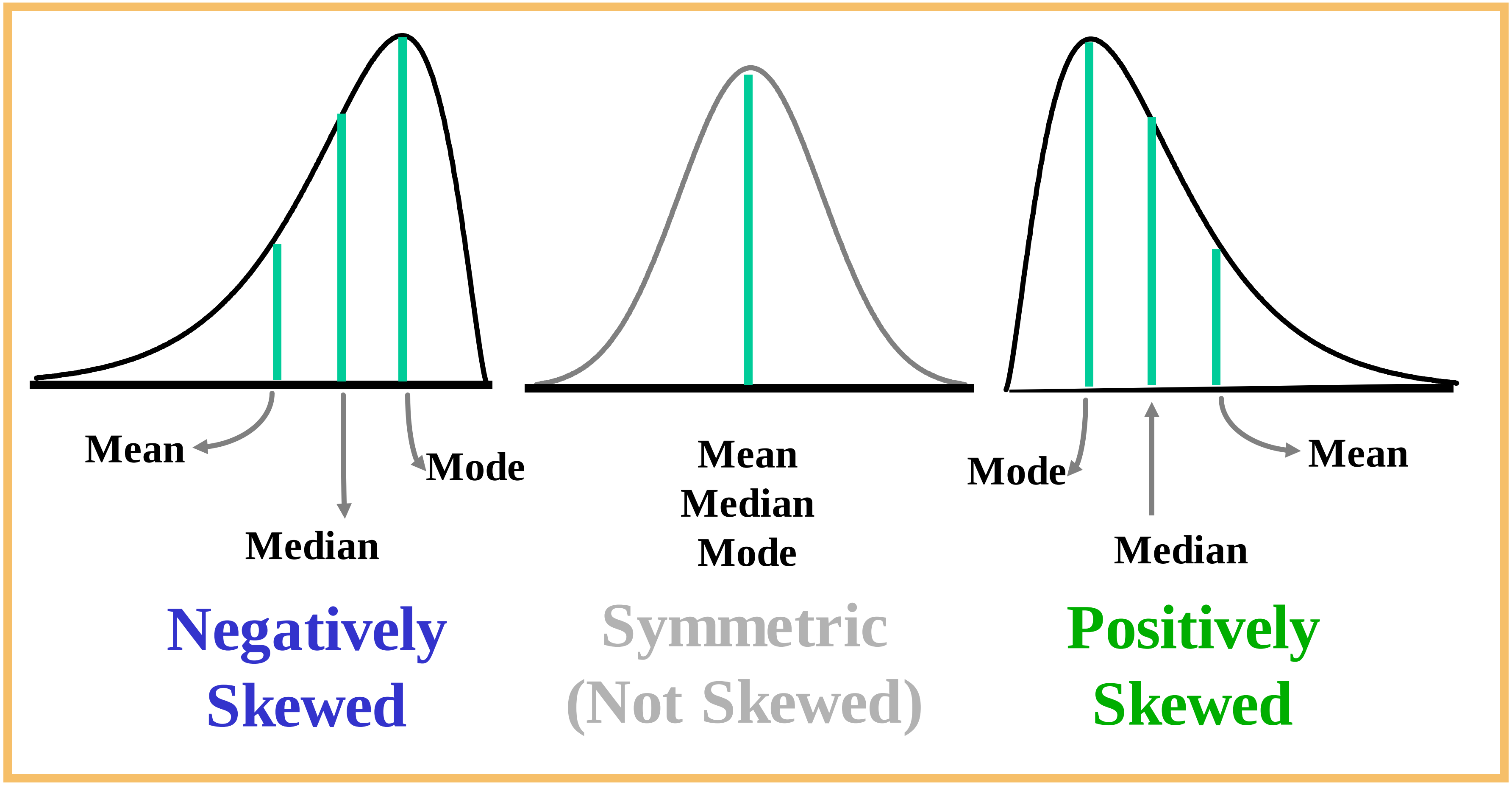
Measures of Shape

- **Skewness**
  - Absence of symmetry
  - Extreme values in one side of a distribution
- **Kurtosis**
  - Peakedness of a distribution
- **Box and Whisker Plots**
  - Graphic display of a distribution
  - Reveals skewness

Skewness

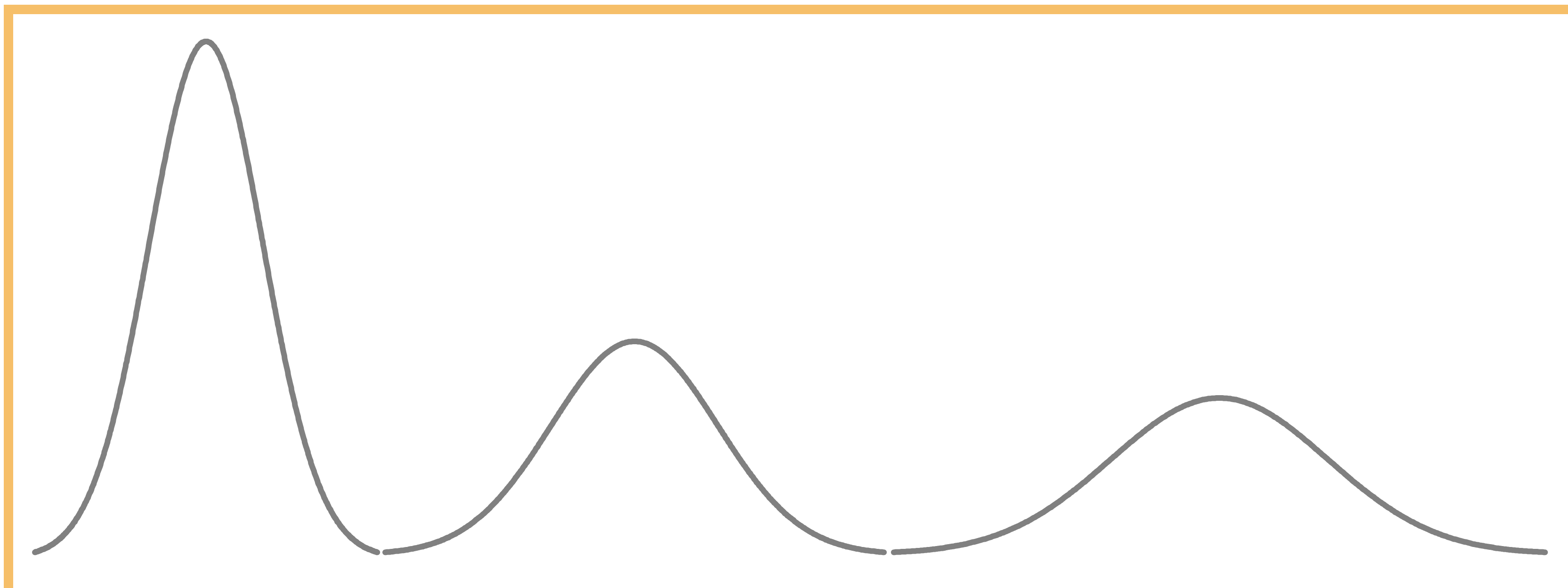

Negatively
Skewed
Symmetric
(Not Skewed)
Positively
Skewed

Skewness



Mean
Median
Mode
Mean
Median
Mode
Mode
Median
Mean
Negatively Skewed
Symmetric (Not Skewed)
Positively Skewed

Kurtosis



Box and Whisker Plot

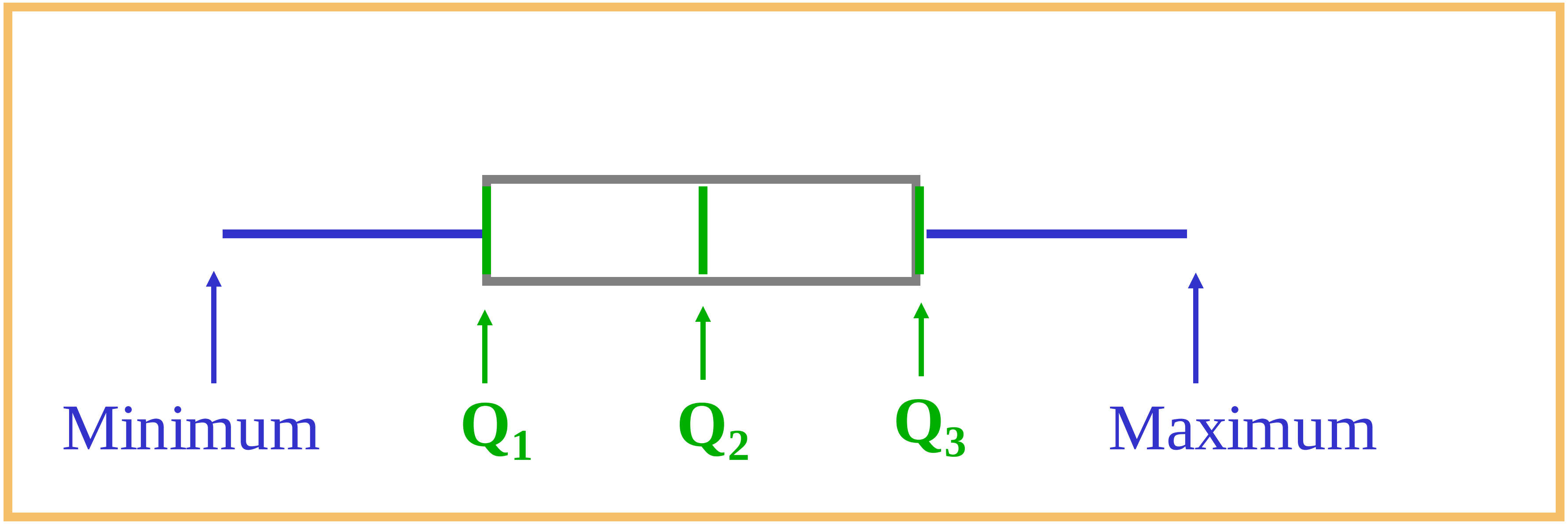
- **Five specific values are used:**
  - **Median, Q_2**
  - **First quartile, Q_1**
  - **Third quartile, Q_3**
  - **Minimum value in the data set**
  - **Maximum value in the data set**

Box and Whisker Plot, *continued*

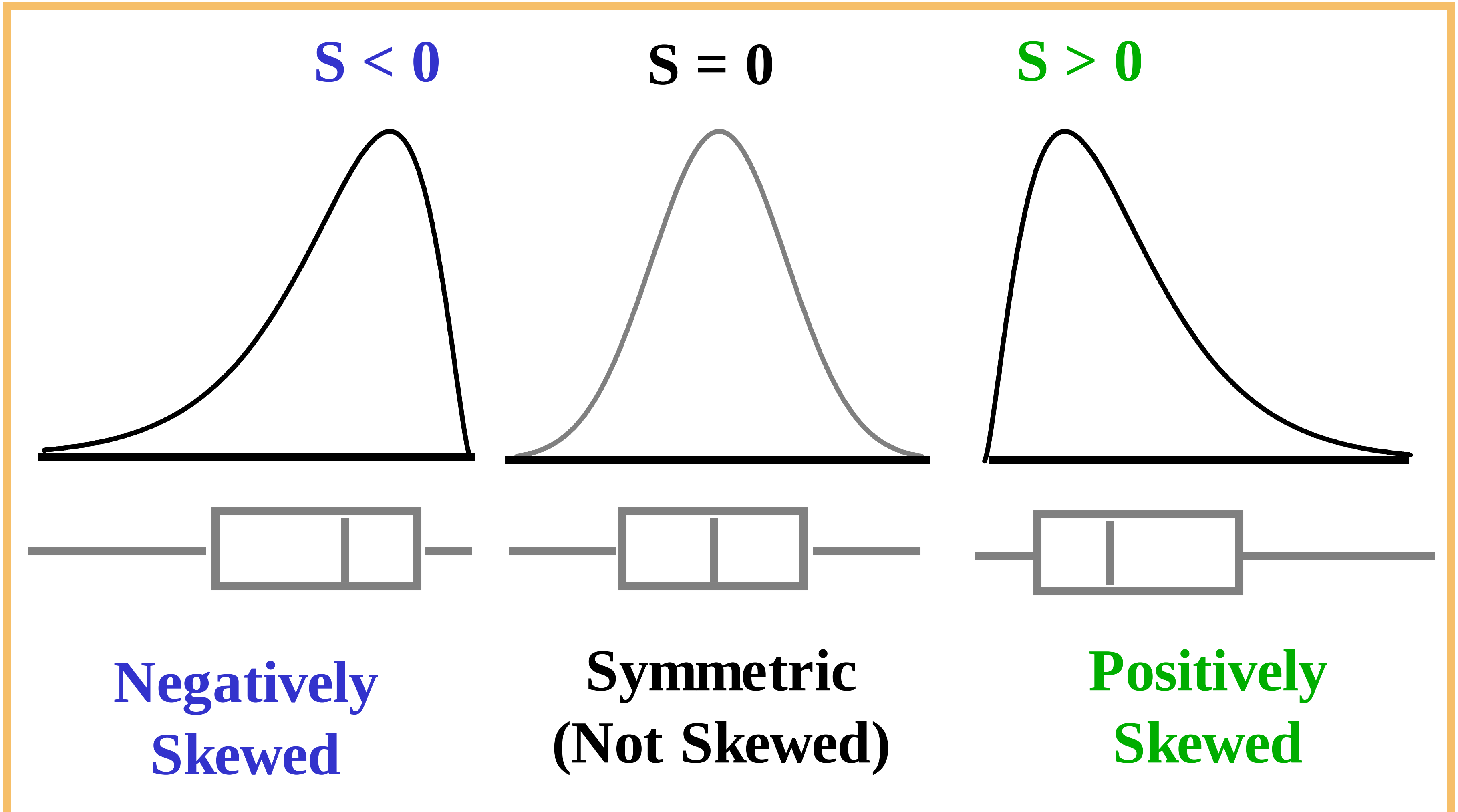
- **Inner Fences**
  - **IQR = Q_3 - Q_1**
  - **Lower inner fence = Q_1 - 1.5 IQR**
  - **Upper inner fence = Q_3 + 1.5 IQR**

- **Outer Fences**
  - **Lower outer fence = Q_1 - 3.0 IQR**
  - **Upper outer fence = Q_3 + 3.0 IQR**

Box and Whisker Plot



Skewness: Box and Whisker



Binomial Distribution

Binomial Experiment

1. The **binomial experiment** consists of a fixed number of trials: n
2. Each trial has two possible outcomes: *success* and *failure.*
3. The probability of success is p. The probability of failure is $1 – p$.
4. The trials are independent.

Binomial random variable is the number of successes in n trials.

Each trial is a **Bernoulli process** if properties 2 – 4 are satisfied.

Binomial experiment?

- Flip a coin 10 times.
- Draw 5 cards out of a shuffled deck.
- A political survey asks 1500 voters whom they intend to vote.

Binomial Probability Distribution

Example: What is the probability of getting 2 heads when a fair coin is flipped 4 times?

Binomial Probability Distribution

Example: What is the probability of getting 2 heads when a fair coin is flipped 4 times?

Solution: *HHTT*, *HTHT*, *HTTH*, *THHT*, *THTH*, *TTHH*

So there are 6 ways to get 2 heads in 4 flips. ($C_2^4 = 6$)

Each sequence has probability $(0.5)^2(0.5)^2$.

P(2 heads in 4 flips) = $6(0.5)^2(0.5)^2 = .375$

Binomial Probability Distribution

x = number of successes in binomial experiment with n trials.
x takes values 0, 1, 2, … , n, therefore, it is discrete.
$n - x$ = number of failures.

Probability that there are x successes and $n - x$ failures:

$$p^x (1-p)^{n-x}$$

Number of ways to get x successes and $n - x$ failures:

$$C_x^n = \frac{n!}{x!(n-x)!}$$

where $n! = n(n-1)(n-2) \cdot \cdot \cdot (2)(1)$, e.g. $0!=1$, $3!=3(2)(1)=6$.

Binomial Probability Distribution

The probability of x successes in a binomial experiment with n trials and probability of success p is

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \text{ for } x = 0, 1, \ldots, n$$

Example: A quiz consists of 10 multiple-choice questions. Each question has 5 possible answers, only one of which is correct. Pat plans to guess the answer to each question. Find the probability that Pat gets

a. one answer correct.

b. all 10 answers correct.

Solution: n = 10, p = .2

a. $$P(1)=\frac{10!}{1!(10-1)!}(.2)^1(1-.2)^{10-1}=10(.2)(.8)^9=.2684$$

b. $$P(10)=\frac{10!}{10!(10-10)!}(.2)^{10}(1-.2)^{10-10}=1(.2)^{10}(1)=.0000001$$

Cumulative Probability

$$P(X \le x) = P(0) + P(1) + \ldots + P(x)$$

Example: Find the probability that Pat fails the quiz. A mark is considered a failure if it is less than 50%.

Solution: A mark of less than 5 is a failure.

$$\begin{aligned} P(X \le 4) &= P(0) + P(1) + P(2) + P(3) + P(4) \\ &= .1074 + .2684 + .3020 + .2013 + .0881 \\ &= .9672 \end{aligned}$$

Using the cumulative probability

$$P(X \geq x) = 1 - P(X \leq x - 1)$$

Example: Find the probability that Pat passes the quiz.

Solution: A mark of 5 or greater is a pass.

$$\begin{aligned} P(X \geq 5) &= 1 - P(X \leq 4) \\ &= 1 - .9672 \\ &= .0328 \end{aligned}$$

Using the cumulative probability

$$P(X = x) = P(X \le x) - P(X \le x - 1)$$

Example: Find the probability that Pat gets one answer correct.

Solution:

$$\begin{aligned} P(1) &= P(X \le 1) - P(X \le 0) \\ &= .3758 - .1074 \\ &= .2684 \end{aligned}$$

Using the cumulative probability

$$P(a \le X \le b) = P(X \le b) - P(X \le a - 1)$$

Example: Find the probability that Pat gets 5 or 6 answers correct.

Solution:

$$\begin{aligned} P(5 \le X \le 6) &= P(X \le 6) - P(X \le 4) \\ &= .9991 - .9672 \\ &= .0319 \end{aligned}$$

(or $P(5 \le X \le 6) = P(5) + P(6) = .0264 + .0055 = .0319$)

Mean and Variance of a Binomial Distribution

$$\mu = np$$
$$\sigma^2 = np(1 - p)$$
$$\sigma = \sqrt{np(1 - p)}$$

Example: If a class is full of students like Pat, what is the mean mark? What is the standard deviation?

Solution:

$$\mu = np = 10(0.2) = 2$$
$$\sigma = \sqrt{np(1 - p)} = \sqrt{10(.2)(1 - .2)} = 1.26$$

Poisson Distribution

Poisson Experiment

1. The number of successes that occur in any interval is independent of the number of successes that occur in any other intervals.
2. The probability of a success in an interval is the same for all equal-size intervals.
3. The probability of a success in an interval is proportional to the size of the interval.
4. The probability of more than one success in an interval approaches 0 as the interval becomes smaller.

Poisson Random Variable

The **Poisson random variable** is the number of successes that occur in a period of time or an interval of space in a Poisson experiment.

As a general rule, a Poisson random variable is the number of *relatively rare* event that occurs *randomly* and *independently*.

Example: (not Poisson random variables)

- The number of hits on an active website
- The number of people arriving at a restaurant

Poisson Random Variable

Example: (Poisson Random Variables)

- The number of cars arriving at a service station in 1 hour
- The number of flaws in a bolt of cloth
- The number of accidents in 1 day on a particular stretch of highway

Poisson Probability Distribution

The probability that a Poisson random variable assumes a value of x is

$$P(x) = \frac{e^{-\mu}\mu^{x}}{x!} \text{ for } x = 0, 1, 2, \ldots$$

where μ is the mean number of successes in the interval or region and e is the base of the natural logarithm (approx. 2.71828).

The variance of the Poisson r.v.

$$\sigma^2 = \mu$$

Example: The number of arrivals at a car wash is Poisson distributed with a mean of 8 per hour. What is the probability that

a. no car will arrive in the next hour?

b. at most 8 cars will arrive in the next hour?

Solution: Use $\mu = 8$.

a. $P(0) = e^{-8}8^{0}/0! = .0003$

b. $P(X \leq 8) = P(0) + P(1) + \ldots + P(8)$

Using cumulative probability in MiniTab gives

$P(X \leq 8) = .5925$

Example: The number of arrivals at a car wash is Poisson distributed with a mean of 8 per hour. What is the probability that

a. no car will arrive in the next 2 hours?

b. at most 10 cars will arrive in the next 2 hours?

c. at least 15 cars will arrive in the next 2 hours?

Solution: The mean number of car arrivals in 2 hour period is 16. Thus, use $\mu = 16$.

a. $P(0) = e^{-16}16^{0}/0! = 0.0000001$

b. $P(X \leq 10) = 0.0774$

c. $P(X \geq 15) = 1 – P(X \leq 14) = 1 – .3675 = .6325$

Example: The number of arrivals at a car wash is Poisson distributed with a mean of 8 per hour. What is the probability that

a. exactly 15 cars will arrive in the next 2 hours?

b. between 10 to 20 cars will arrive in the next 2 hours?

Solution: $\mu = 16$.

a. $P(15) = P(X \leq 15) - P(X \leq 14)$

$= .4667 - .3675 = .0992$

b. $P(10 \leq X \leq 20) = P(X \leq 20) - P(X \leq 9)$

$= .8682 - .0433 = .8249$

Recall $P(a \leq X \leq b) = P(X \leq b) - P(X \leq a - 1)$

Normal Distributions

Learning Objectives

- Interpret graphs of normal probability distributions
- Find areas under the standard normal curve

Properties of Normal Distributions

Normal distribution

- A continuous probability distribution for a random variable, *x*.
- The most important continuous probability distribution in statistics.
- The graph of a normal distribution is called the **normal curve**.

Properties of Normal Distributions

1. The mean, median, and mode are equal.
2. The normal curve is bell-shaped and is symmetric about the mean.
3. The total area under the normal curve is equal to 1.
4. The normal curve approaches, but never touches, the *x*-axis as it extends farther and farther away from the mean.

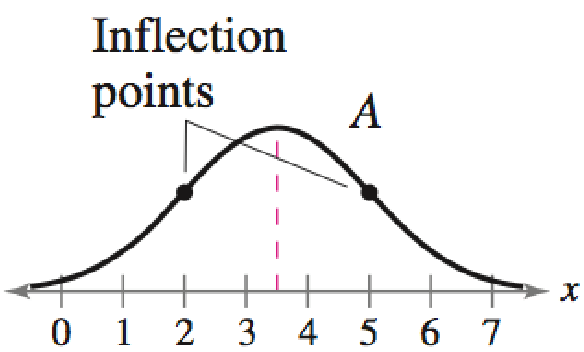
Properties of Normal Distributions

5. Between $\mu - \sigma$ and $\mu + \sigma$ (in the center of the curve), the graph curves downward. The graph curves upward to the left of $\mu - \sigma$ and to the right of $\mu + \sigma$. The points at which the curve changes from curving upward to curving downward are called the **inflection points**.

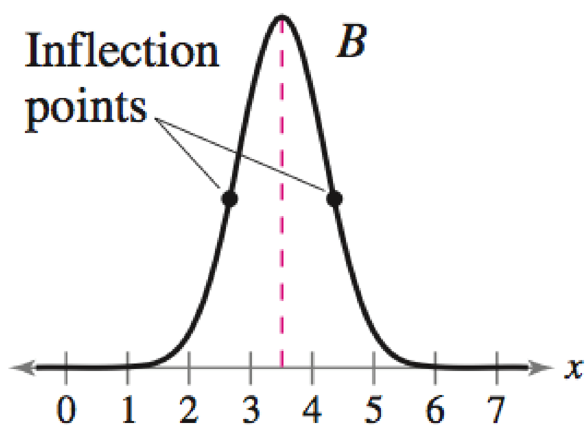
Means and Standard Deviations

- A normal distribution can have any mean and any positive standard deviation.
- The mean gives the location of the line of symmetry.
- The standard deviation describes the spread of the data.

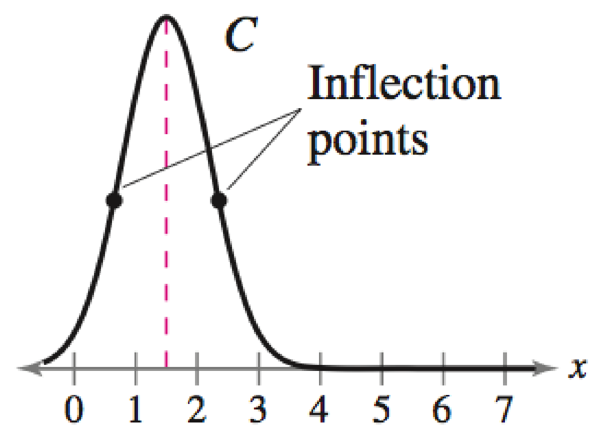
Means and Standard Deviations



$\mu = 3.5$
$\sigma = 1.5$



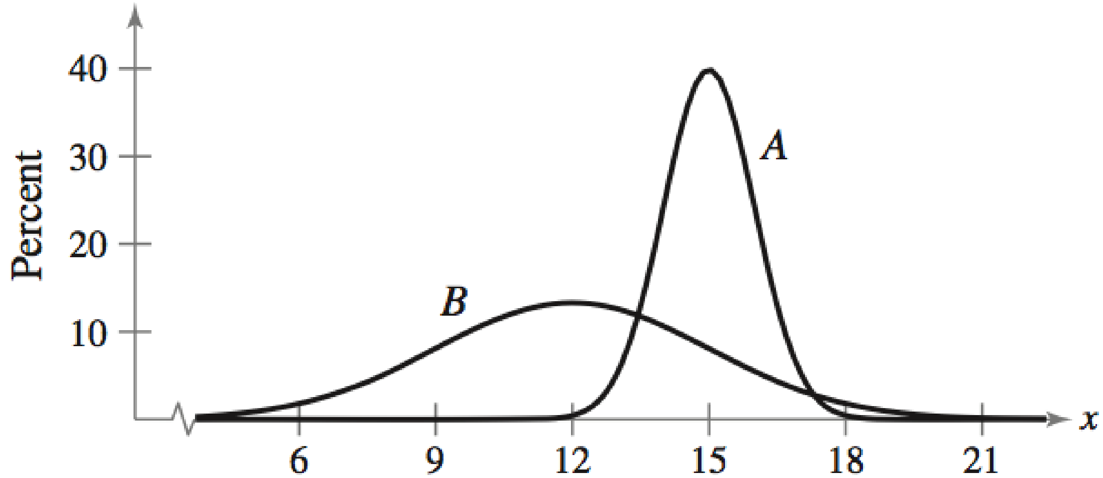
$\mu = 3.5$
$\sigma = 0.7$



$\mu = 1.5$
$\sigma = 0.7$

Example: Understanding Mean and Standard Deviation

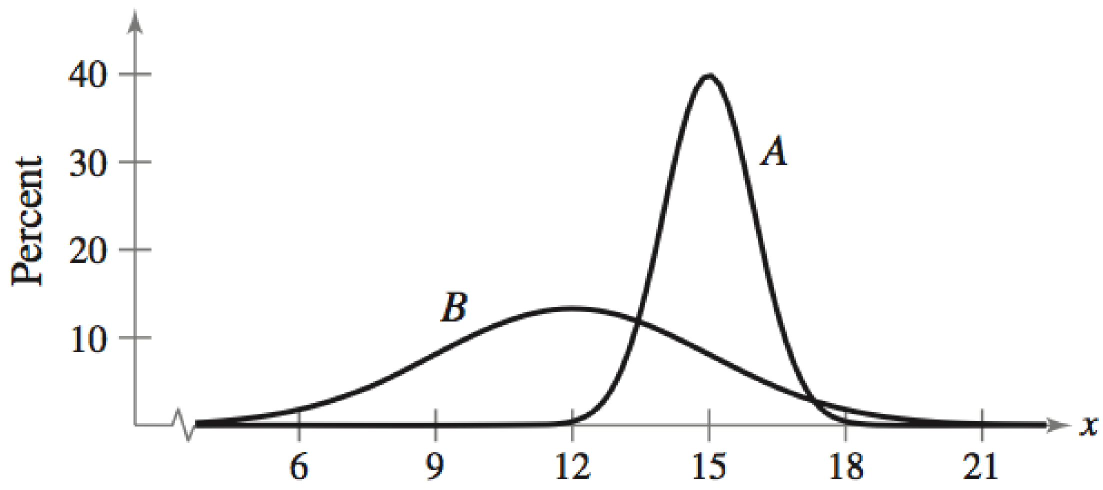
1. Which normal curve has the greater mean?



Curve *A* has the greater mean (The line of symmetry of curve *A* occurs at x = 15. The line of symmetry of curve *B* occurs at x = 12.)

Example: Understanding Mean and Standard Deviation

2. Which curve has the greater standard deviation?



Curve *B* has the greater standard deviation (Curve *B* is more spread out than curve *A*.)

The Standard Normal Distribution

Standard normal distribution

- A normal distribution with a mean of 0 and a standard deviation of 1.

- Any *x*-value can be transformed into a *z*-score by using the formula

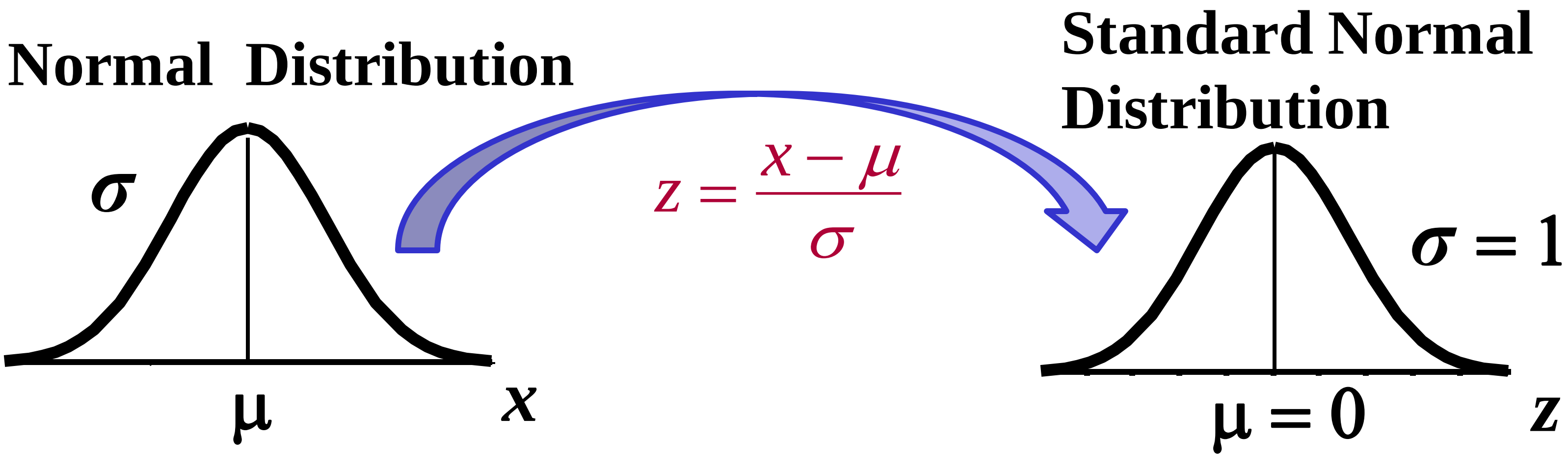
$$z = \frac{\text{Value} - \text{Mean}}{\text{Standard deviation}} = \frac{x - \mu}{\sigma}$$

The Standard Normal Distribution

- If each data value of a normally distributed random variable x is transformed into a z-score, the result will be the standard normal distribution.

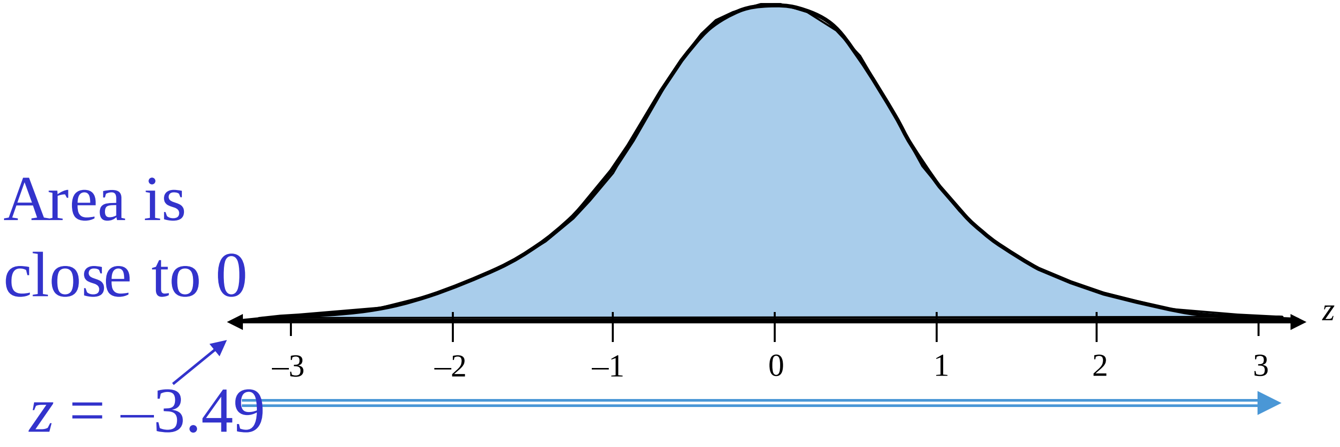
- Use the Standard Normal Table to find the cumulative area under the standard normal curve.

The Standard Normal Distribution



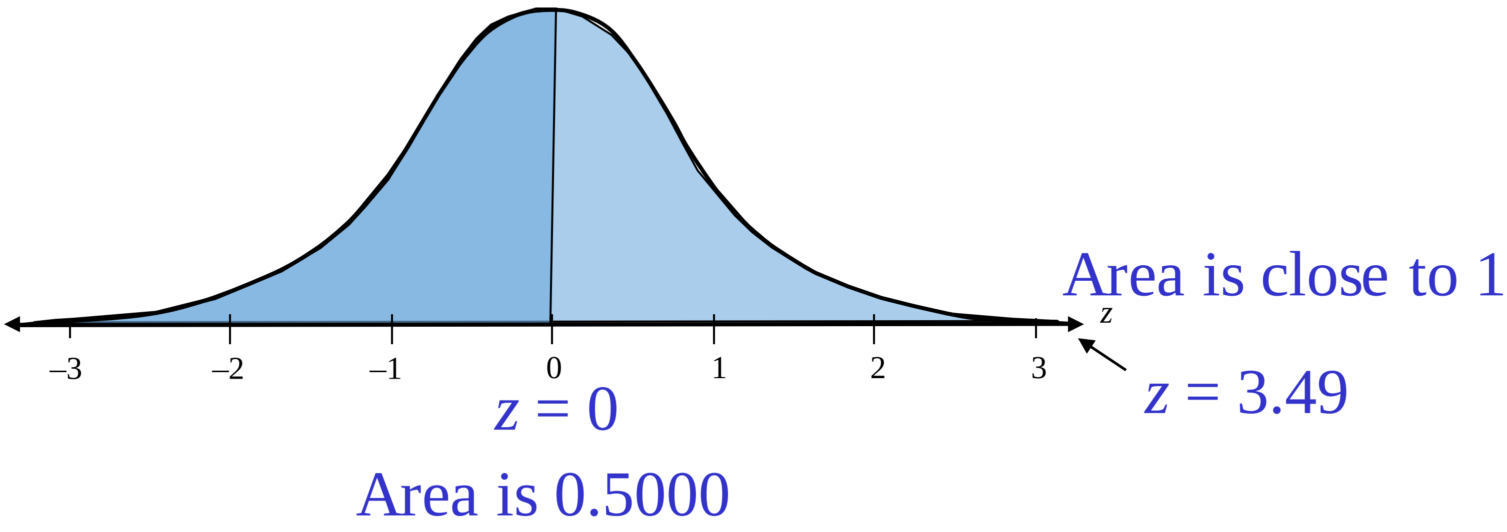
Properties of the Standard Normal Distribution

1. The cumulative area is close to 0 for *z*-scores close to $z = -3.49$.
2. The cumulative area increases as the *z*-scores increase.



Properties of the Standard Normal Distribution

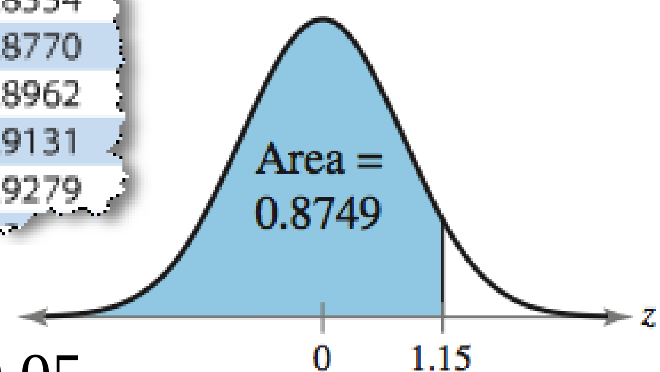
3. The cumulative area for $z = 0$ is 0.5000.
4. The cumulative area is close to 1 for z-scores close to $z = 3.49$.



Example: Using The Standard Normal Table

Find the cumulative area that corresponds to a *z*-score of 1.15.

z	.00	.01	.02	.03	.04	.05	.06
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279



Find 1.1 in the left hand column.

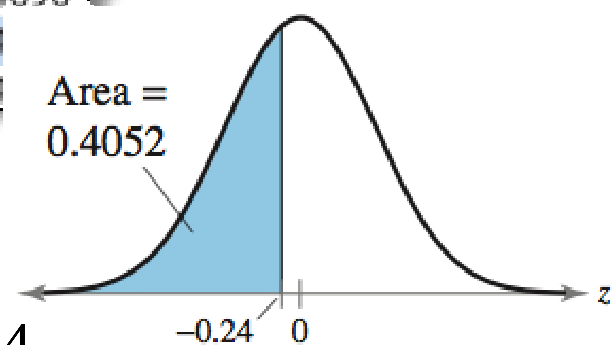
Move across the row to the column under 0.05

The area to the left of $z = 1.15$ is 0.8749.

Example: Using The Standard Normal Table

Find the cumulative area that corresponds to a *z*-score of –0.24.

z	.09	.08	.07	.06	.05	.04	.03
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006
−0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981
−0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4
−0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4



Solution:

Find –0.2 in the left hand column.

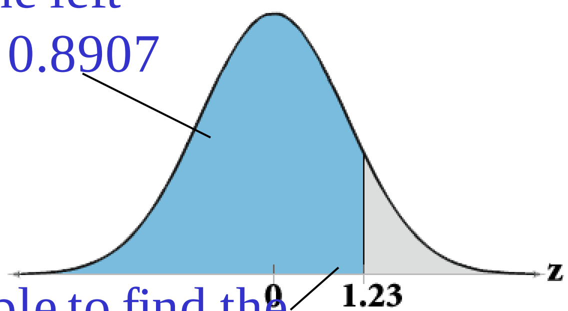
Move across the row to the column under 0.04

The area to the left of *z* = –0.24 is 0.4052.

Finding Areas Under the Standard Normal Curve

1. Sketch the standard normal curve and shade the appropriate area under the curve.
2. Find the area by following the directions for each case shown.
   a. To find the area to the *left* of z, find the area that corresponds to z in the Standard Normal Table.

2. The area to the left of $z = 1.23$ is 0.8907



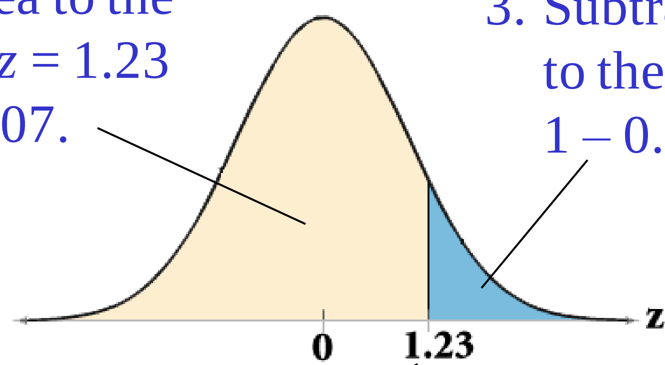
1. Use the table to find the area for the z-score

Finding Areas Under the Standard Normal Curve

b. To find the area to the *right* of *z*, use the Standard Normal Table to find the area that corresponds to *z*. Then subtract the area from 1.

2. The area to the left of $z = 1.23$ is 0.8907.

3. Subtract to find the area to the right of $z = 1.23$: $1 - 0.8907 = 0.1093$.



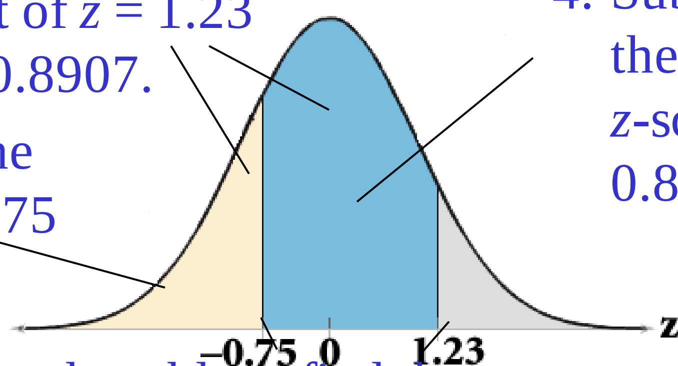
1. Use the table to find the area for the *z*-score.

Finding Areas Under the Standard Normal Curve

c. To find the area *between* two z-scores, find the area corresponding to each z-score in the Standard Normal Table. Then subtract the smaller area from the larger area.

2. The area to the left of $z = 1.23$ is 0.8907.

3. The area to the left of $z = –0.75$ is 0.2266.

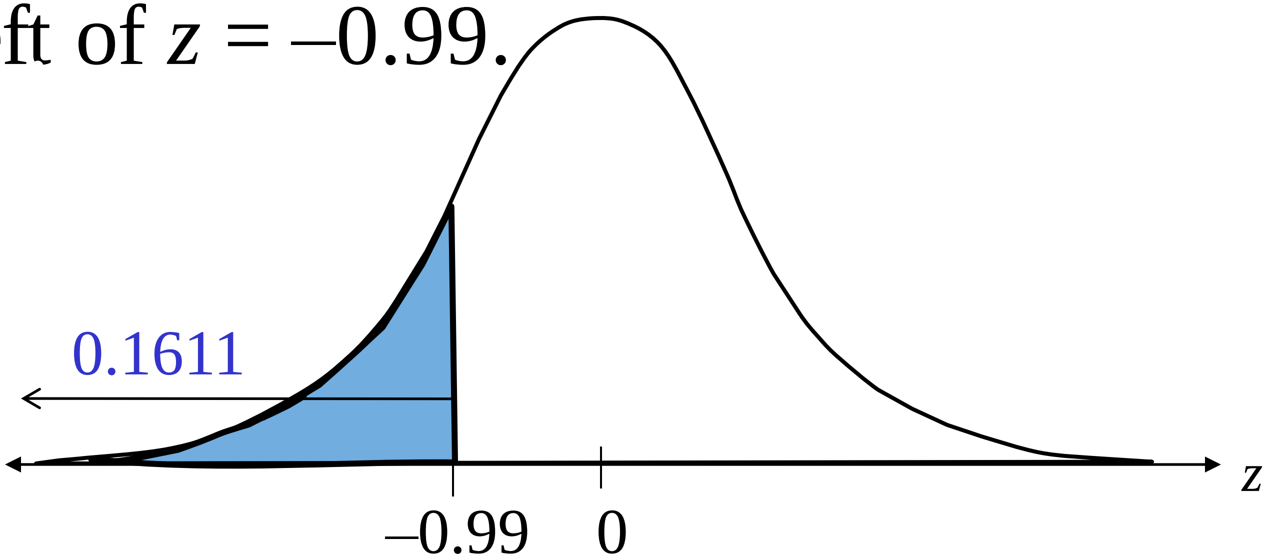


4. Subtract to find the area of the region between the two z-scores: $0.8907 – 0.2266 = 0.6641$.

1. Use the table to find the area for the z-scores.

Example: Finding Area Under the Standard Normal Curve

Find the area under the standard normal curve to the left of $z = -0.99$.

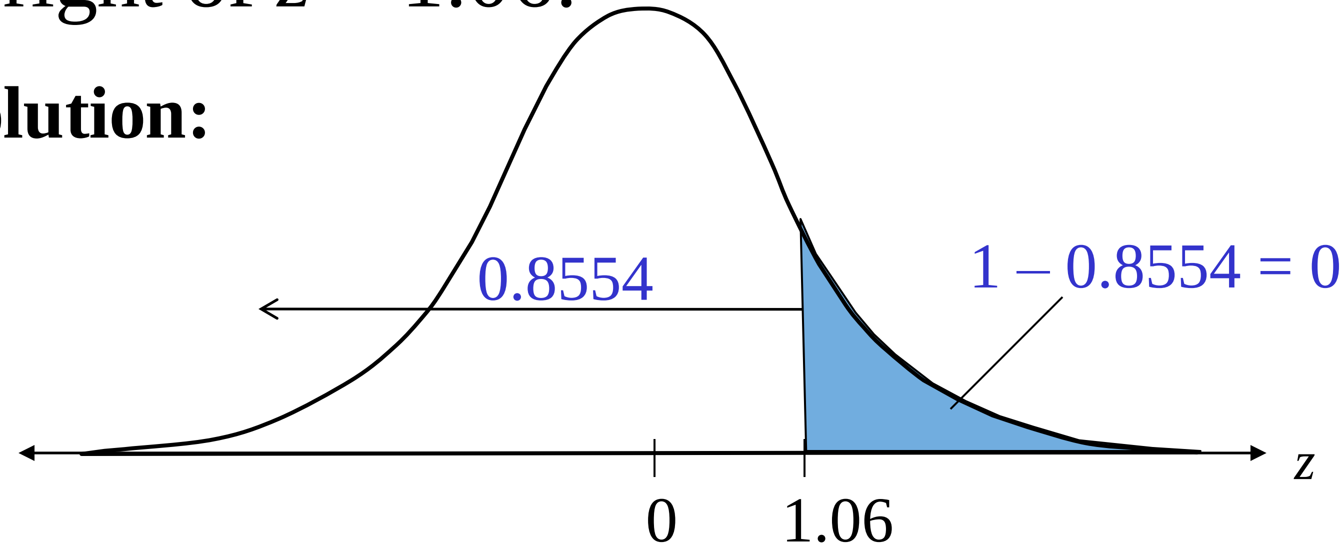


From the Standard Normal Table, the area is equal to 0.1611.

Example: Finding Area Under the Standard Normal Curve

Find the area under the standard normal curve to the right of $z = 1.06$.

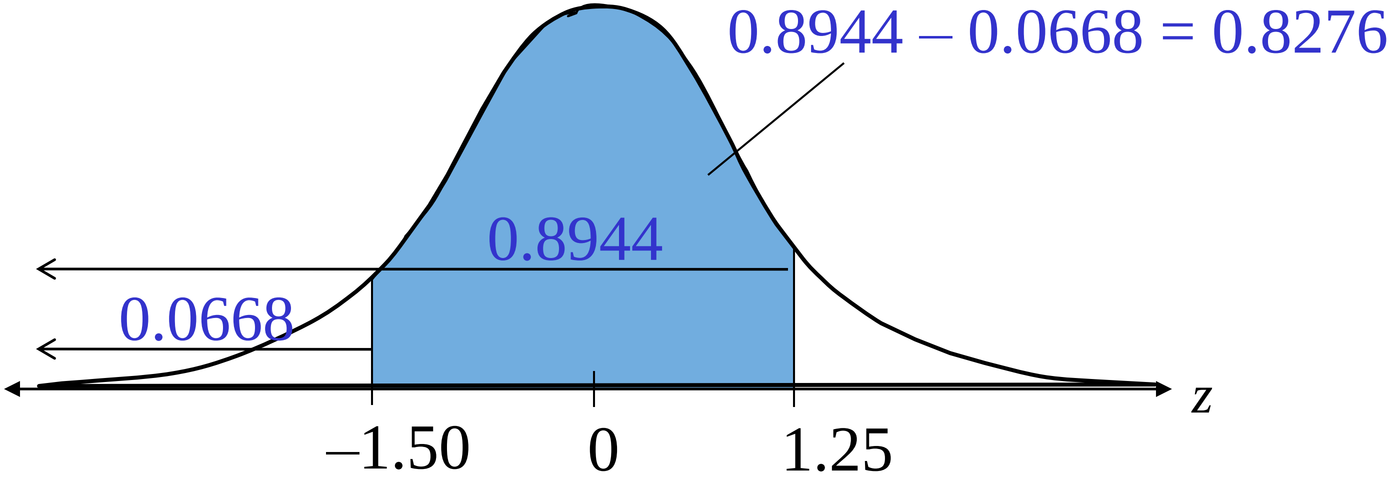
Solution:



From the Standard Normal Table, the area is equal to 0.1446.

Example: Finding Area Under the Standard Normal Curve

Find the area under the standard normal curve between $z = -1.5$ and $z = 1.25$.



From the Standard Normal Table, the area is equal to 0.8276.